DETERMINATION OF THERMAL DIFFUSIVITY UNDER MIXED BOUNDARY CONDITIONS

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An examination is made of different variants of the determination of thermal diffusivity when boundary conditions fixed in time are imposed. It is shown that for these cases the thermal diffusivity may be determined by a single method.

Under mixed boundary conditions, by which we understand a combination of boundary conditions of the first and second or of the second and third kinds, the final objective in the majority of cases is to determine the thermal diffusivity by an absolute or a comparatively stationary method. Use of an unsteady thermal state preceding a steady state permits us, even in this case, to introduce at least two constants, such as the cooling rate [1], knowledge of which allows us always to determine the thermal diffusivity. Before considering these quantities, we shall turn to the case when boundary conditions of the first kind prevail on the surface of a specimen.

The theoretical basis for determination of thermal diffusivity may be any solution [2] with boundary conditions of the first or of the third kind.

We shall consider one of the possible variants of methods for determining the thermal diffusivity of solids in the form of plates or discs. The expression for the relative dimensionless temperature at the center of a plate whose surface is maintained at a constant temperature has the following form [2]:

$$\Theta_c = \frac{t(x,\ \tau) - t_c}{t_0 - t_c} = \sum_{n=1}^{\infty} A_n \exp(-\mu_n^2 \mathrm{Fo}). \qquad (1)$$

When Fo ≥ 0.2, the temperature field of a plate is described by the first term of the series (1), to a high degree of accuracy. Therefore

$$\Theta_c = \frac{2}{\mu_1} \exp(-\mu_1^2 \mathrm{Fo}) \quad \left(\mu_1 = \frac{\pi}{2}\right). \qquad (2)$$

It follows from (2) that

$$a = \frac{4R^2}{\pi^2} \frac{\ln \Theta_1 - \ln \Theta_2}{\tau_2 - \tau_1}. \qquad (3)$$

Relation (3) differs from the corresponding relation in regular regime theory only as regards the coefficient. The coefficient appearing in (3) could be called the shape coefficient of an infinite plate. However, in our opinion, this term is not entirely fortunate. In fact, in the case, for example, of a multi-layer plate, the shape and the nature of the variation of the temperature field are retained, evidently, as before, but the coefficient of (3) will be different.

It is not difficult to show that the value of the cooling rate in (3) may be replaced in principle by another quantity, which, like the first, is a constant. In fact, at the stage of an established (regular) thermal process

$$\Theta = AU \exp(-m\tau),$$

$$\frac{d\Theta_1}{d\tau_1} = b_1 = -mAU \exp(-m\tau_1),$$

$$\frac{d\Theta_2}{d\tau_2} = b_2 = -mAU \exp(-m\tau_2),$$

$$\frac{\ln \Theta_1 - \ln \Theta_2}{\tau_2 - \tau_1} = \frac{\ln b_1 - \ln b_2}{\tau_2 - \tau_1} = m.$$

The meaning of the substitution becomes clear when an analysis is made of solutions with combined boundary conditions.

We shall examine the solution of the problem of heating of one surface of an infinite plate by a constant heat flux under the condition that the other surface is maintained at constant temperature [3]:

$$\Theta_c = t_c - t_0 = \frac{qR}{\lambda} - 2\frac{qR}{\lambda} \sum_{n=1}^{\infty} \frac{\exp(-\mu_n^2 \mathrm{Fo})}{\mu_n^2}.$$

When Fo > 0.3

$$\Theta_c = \frac{qR}{\lambda} - 2\frac{qR}{\lambda\mu_1^2} \exp(-\mu_1^2 \mathrm{Fo}). \qquad (4)$$

It follows from (4) that

$$\frac{\ln b_1 - \ln b_2}{\tau_2 - \tau_1} = \frac{\pi^2}{4R^2} a = m, \qquad (5)$$

$$a = \frac{4R^2}{\pi^2} \frac{\ln b_1 - \ln b_2}{\tau_2 - \tau_1}. \qquad (6)$$

It is clear from (5) that the new quantity (it could be called the rate of change of heating rate) is constant even under combined boundary conditions. It should be stressed that the fact that this quantity is constant not only for the special examples examined but in general for all cases for which the end result of the development of the temperature field is a steady thermal state.

Determination of the thermal conductivity on the basis of (6) assumes the need to know the nature of the variation of the temperature field $t = f(\tau)$ and requires subsequent processing of this quantity in order to construct the graph $\ln b = \varphi(\tau)$ (theoretically a straight line). However, determination of the rate of heating is associated with substantial errors. For this reason it is preferable to use another variant for the calculation of thermal diffusivity, which, other conditions being equal, leads to smaller errors than the calculation on the basis of (6). In fact, the first term on the right side of (4) is none other than $t_{c(max)} - t_0 = \Theta_{c(max)} = qR/\lambda$, i.e., it is the maximum temperature difference which is established between the center and the base of the plate in the steady state. Therefore

$$\Theta_{c(max)} - \Theta = t_{c(max)} - t_0 = \frac{2}{\mu_1^2} \frac{qR}{\lambda} \exp(-\mu_1^2 \mathrm{Fo}). \quad (7)$$

From (7)

$$a = \frac{4R^2}{\pi^2} \frac{\ln(t_{c(max)} - t_{c_1}) - \ln(t_{c(max)} - t_{c_2})}{\tau_2 - \tau_1}. \quad (8)$$

The quantity

$$m' = \frac{\ln(t_{c(max)} - t_{c_1}) - \ln(t_{c(max)} - t_{c_2})}{\tau_2 - \tau_1},$$

appearing in (8), possesses all the properties of the cooling rate.

Similar computational formulas may be obtained also for samples of other geometrical shapes (solid, hollow cylinder, and sphere) with the one difference that the coefficients in (8) will be determined by the roots of the corresponding characteristic equations.

To verify the calculation relations (3), (6), and (8), a simple experimental scheme was used. Test specimens were formed in the shape of plane-parallel square plates or disks. To create the required boundary conditions of the first kind we used a clamp consisting in the main of two hollow plane-parallel cylindrical copper "coolers" with carefully ground surfaces. The coolers were joined together by rubber hoses through which water was passed. One junction of a differential thermocouple was embedded in the surface of one of the blocks beforehand, the second junction being installed at the center of the plate before the test. The temperature difference between the specimen, initially at room temperature, and the "coolers", through which water was flowing, was recorded with a galvanometer. When the maximum steady temperature difference was established, the specimen was placed in the space between the "coolers", and was squeezed tightly between them. The variation of temperature with time was recorded in one case with a galvanometer and stop watch, and in another case with a photo-electric recorder type N 373-1. The treatment of the results obtained to find the thermal conductivity on the basis of (3) was carried out by the method usually applied in the regular regime theory. In doing this, in order to avoid the need to construct a graph of $\ln\Theta = f(\tau)$, we used another method, the essence of which was to record the time interval during which the temperature difference changed by a specific fraction of its initial value. Since the specimen is located initially inside the "coolers", through which a coolant is passing, the temperature difference between them is always known. For example, let the time interval during which the temperature difference decreases from 1/2 to 1/4 of its initial value be noted during the experiment; it then follows from (3) that

$$a = \frac{4R^2}{\pi^2} \frac{\ln 2}{\tau_2 - \tau_1}. \quad (9)$$

The data of [2] allow choice of just such an interval of temperature change, in which regular thermal process occurs. The combined boundary conditions are created by locating a low-inertia plane-parallel heater at the center of the plate (the plate is composite). The experimental layout is practically no different from that described.

The test specimen is compressed in a clamp between plane "coolers" through which, as in the first case, water at constant temperature is passed. A constant electrical power is supplied to the heater. From the experiment we find the relation $t = f(\tau)$ and t_{max} (in the steady state). From these data a graph of $\ln(t_{max} - t) = \varphi(t)$ is constructed, which is a straight line for an established thermal process (Fo > 0.3). Knowledge of the slope of this line permits us to calculate the thermal diffusivity from relation (8).

We investigated specimens of plexiglass and of certain other materials. For plexiglass, which we chose as a "standard" material, the mean value of the thermal diffusivity found from relations (3) and (8), was m^2/sec, which agreed with the available data in the literature within 3–6%. The maximum error in determining the thermal diffusivity on the basis of (6) was 10–12%.

We restricted ourselves in this case to a comparison of the results obtained with results of other authors [4], which in essence is only an indirect evaluation of the accuracy of the techniques examined.

The error in determining thermophysical properties by any method, as is known, resolves into two parts, of which one part is determined by the error in measuring the quantity entering into the calculation formulas. Evaluation of this part of the error is not difficult, and the quantity may be reduced to a minimum by using a more refined measurement technique. The greatest difficulty is to evaluate that part of the total error which is due to deviation of the conditions of the experiment from the required theoretical prerequisites.

Some of the assumptions and calculations that we made in estimating the second part of the error are as follows. Relations (3) and (8) were obtained from one-dimensional solutions. In order to justify the validity of applying them to specimens of finite size, we carried through calculations based on solutions of the corresponding two-dimensional problems. These calculations provided a basis for reaching the conclusion that if the ratio between the linear dimensions of the plate is equal to or greater than 1/3, the nature of the variation of the temperature field in the measurement region (the center of the plate) agrees with the corresponding values in the infinite plate to within 0.5%.

It is clear that these inferences will be valid only for the case when the temperature difference between the specimen and the surrounding medium is not too large.

The second appreciable and difficult to calculate source of error is the additional thermal resistances arising at the points of contact of the plate with the plane "coolers". In view of the absence of any other general criteria permitting us to estimate the error of this type, we took steps to reduce it by careful treatment of the surface and by compressing the specimen. It was assumed also that change in the thermophysical properties α, λ, C for small temperature drops does not introduce distortion into the nature of variation of the temperature field, and, therefore, avoids the need to introduce any correction coefficients into the calculation formulas.

The method that we used to determine the thermal diffusivity of solid materials, subject to boundary conditions of the first kind, is simpler than the calorimeter method; moreover, the imposition of combined boundary conditions allows a composite determination of the thermophysical properties to be done, using a combination of purely unsteady and steady thermal states.

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